মৌল কক্ষ

আমি পালো $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$= (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

$= a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

য'ত C_{ij} বোৰ হ'ল a_{ij} ৰ সহৰাশি।

4.6 Adjoint and Inverse of a matrix:
(সহগুন্থি আৰু মৌলকক্ষৰ প্ৰতিলোম)

ধৰা হওক $A = (a_{ij})_{n\times n}$ এটা বৰ্গ মৌলকক্ষ।

(1) তেন্তে আৰু a_{ij} ৰ সহৰাশি A_{ij}, তেনেক্ষেত্ৰত A_{ij} সমূহক মৌল হিচাপে লৈ পোৱা $[A_{ij}]_{n\times n}$ মৌলকক্ষক পক্ষান্তৰ কৰি পোৱা মৌলকক্ষক A মৌলকক্ষৰ সহগুন্থি বা সহযুক্ত বোলে আৰু ইয়াক adj A হিচাপে লিখা হয়।

ধৰা হ'ল $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ তেন্তে $\text{Adj}\,A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$

$= \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

Q $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ ৰ adj A উলিওৱা।

Soln ইয়াত 2 ৰ সহৰাশি $= 4 = A_{11}$

3 ৰ " $= A_{12} = -1$

1 ৰ " $= A_{21} = -3$

4 ৰ " $= A_{22} = 2$

$\therefore \text{adj}\,A = \begin{pmatrix} 4 & -1 \\ -3 & 2 \end{pmatrix}' = \begin{pmatrix} 4 & -3 \\ -1 & 2 \end{pmatrix}$ Ans.

2×2 matrix ৰ সহৰাশি-সহজে উলিয়া

Note: প্ৰথমে A ৰ পক্ষান্তৰ কৰি তাৰ পাছত সহৰাশি বহুৱালেও একেই উঃ পোৱা যাব।

৪ তলৰ মেট্ৰিক্সটোৰ সহগুণ্ডি (adjoint) উলিওৱা।

$$A = \begin{pmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{pmatrix}$$

~~উঃ তে $A_{11} = (-1)^{1+1} \begin{vmatrix} -4 & 2 \\ -1 & 5 \end{vmatrix} = -20+2 = 18$~~

উত্তৰ 2ৰ সহৰাশি $= A_{11} = +\begin{vmatrix} -4 & 2 \\ -1 & 5 \end{vmatrix} = -20+2 = -18$

3 ৰ সহৰাশি $= A_{12} = -\begin{vmatrix} 0 & 2 \\ 1 & 5 \end{vmatrix} = 0-2 = -2$

-4 ৰ " $= A_{13} = +\begin{vmatrix} 0 & -4 \\ 1 & -1 \end{vmatrix} = 0+4 = 4$

0 ৰ " $= A_{21} = -\begin{vmatrix} 3 & -4 \\ -1 & 5 \end{vmatrix} = 15-4 = 11$

-4 ৰ " $= A_{22} = +\begin{vmatrix} 2 & -4 \\ 1 & 5 \end{vmatrix} = 10+4 = 14$

2 ৰ " $= A_{23} = -\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -(-2-3) = 5$

(2) 1 ৰ " $= A_{31} = +\begin{vmatrix} 3 & -4 \\ -4 & 2 \end{vmatrix} = 6-16 = -10$

-1 ৰ " $= A_{32} = -\begin{vmatrix} 2 & -4 \\ 0 & 2 \end{vmatrix} = -4$

5 ৰ " $= A_{33} = +\begin{vmatrix} 2 & 3 \\ 0 & -4 \end{vmatrix} = -8$

$$\therefore Adj\,A = \begin{pmatrix} -18 & -2 & 4 \\ 11 & 14 & 5 \\ -10 & -4 & -8 \end{pmatrix}^T$$

$$= \begin{pmatrix} -18 & 11 & -10 \\ -2 & 14 & -4 \\ 4 & 5 & -8 \end{pmatrix}$$

Calculations ভুল থাকিব পাৰে! চাই ল'লে ভাল

Ex. $A = \begin{bmatrix} 1 & 4 & 5 \\ 3 & 2 & 6 \\ 0 & 1 & 0 \end{bmatrix}$ মৌলিকক্ষৰ Adj A উলিওৱা।

উঃ তে, $A_{11} = -6$, $A_{12} = 0$, $A_{13} = 3$

$A_{21} = 5$, $A_{22} = 0$, $A_{23} = -1$

$A_{31} = 14$, $A_{32} = 9$, $A_{33} = -10$

$$adj\,A = \begin{bmatrix} -6 & 0 & 3 \\ 5 & 0 & -1 \\ 14 & 9 & -10 \end{bmatrix}^T = \begin{bmatrix} -6 & 5 & 14 \\ 0 & 0 & 9 \\ 3 & -1 & -10 \end{bmatrix}$$ #

Thm: A এটা n মাত্ৰাৰ বৰ্গ মৌলকক্ষ হ'লে,

$$A\cdot(\text{adj}A) = (\text{adj}A)A = |A|I.$$

য'ত I হ'ল n মাত্ৰাৰ একক মৌলকক্ষ।

ব্যাখ্যা: $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$\Rightarrow \text{adj}A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

Now, $A\cdot(\text{adj}A) = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix} = |A|\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = |A|I.$

একেদৰে, দেখুৱাব পাৰি যে

$$(\text{adj}A)A = |A|I.$$

(3)

সংজ্ঞা: যদি A এটা বৰ্গাকাৰ মৌলকক্ষ আৰু $|A|=0$, তেনেহ'লে A ক অপ্ৰতিম মৌলকক্ষ (Singular matrix) বুলি কোৱা হয়।

যদি $|A|\neq 0$, তেনেহ'লে A ক অনপ্ৰতিম মৌলকক্ষ (non singular matrix) বোলে।

Ex $A = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$ হ'লে $|A| = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 12-12 = 0$

$\therefore$ A অপ্ৰতিম মৌলকক্ষ (চিংগুলাৰ)

Ex $B = \begin{pmatrix} 3 & 5 \\ 1 & 7 \end{pmatrix}$ হ'লে $|B| = \begin{vmatrix} 3 & 5 \\ 1 & 7 \end{vmatrix} = 21-5 = 16 \neq 0$

B এটা অনপ্ৰতিম মৌলকক্ষ।

Thm এটা মৌলকক্ষ প্ৰতিলোমনীয় হয় যদি আৰু যদিহে এইটো অনপ্ৰতিম (non-singular).

Proof ধৰা হ'ল A এটা n মাত্ৰাৰ প্ৰতিলোমনীয় মৌলকক্ষ। তেনেহ'লে আন এটা মৌলকক্ষ B পোৱা যাব যাতে

$$AB = BA = I.$$

$$\Rightarrow |AB| = |BA| = |I|$$

or, $|A|\,|B| = 1$ $\quad \because |AB| = |A|\,|B|,\ |I| = 1$

$\Rightarrow |A| \neq 0$ $\quad \therefore$ A টো নন-চিংগুলাৰ।

বিপৰীতক্ৰমে, ধৰো A non-singular.

$\Rightarrow |A| \neq 0$

Now, $A \cdot adj A = (adj A) A = |A| I$. ——— ①

$\Rightarrow A\left(\frac{1}{|A|} adj A\right) = \left(\frac{1}{|A|} adj A\right) = I$.

$\Rightarrow AB = BA = I$. $\quad$ যত $B = \frac{1}{|A|} adj A$ ধৰা হৈছে।

$\Rightarrow B = \frac{1}{|A|} adj A$.

$\Rightarrow$ A প্ৰতিলোমনীয় আৰু $A^{-1} = \frac{1}{|A|} adj A$.

(4) Ex. যদি $A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$ এটা মৌলকক্ষ হ'লে A^{-1} উলিওৱা।

Step 1: A এটা non singular মৌলকক্ষ:

$$|A| = \begin{vmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{vmatrix} = 1(16-9) - 3(4-3) + 3(3-4) = 1 \neq 0$$

$\therefore$ A মৌলকক্ষটো অনপ্ৰতিম।

আকৌ সহৰাশিবোৰ $A_{11} = 7,\ A_{12} = -1,\ A_{13} = -1$

$A_{21} = -3,\ A_{22} = 1,\ A_{23} = 0$

$A_{31} = -3,\ A_{12} = 0,\ A_{33} = 1$

Therefore, $adj A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

এতিয়া, $A \cdot (adj A) = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 7-3-3 & -3+3+0 & -3+0+3 \\ 7-4-3 & -3+4+0 & -3+0+3 \\ 7-3-4 & -3+3+0 & -3+0+4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = (1)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = |A| I.$$

একেদৰে $A \cdot (adj A) = (adj A) A = |A| \cdot I$.

$\therefore A^{-1} = \frac{1}{|A|} \operatorname{adj} A$.

$$= \frac{1}{1}\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}.$$

Ex. $A = \begin{pmatrix} 8 & 4 & 2 \\ 2 & 9 & 4 \\ 1 & 2 & 8 \end{pmatrix}$ হ'লে A^{-1} উলিওৱা।

Soln

$$|A| = 8\times(72-8) - 4(16-4) + 2(4-9) = 454 \neq 0$$

$\therefore$ A মৌলকক্ষটো নন চিংগুলাৰ (অনপ্ৰতিম). সেতিকে A^{-1} থাকে।

এতিয়া, $A_{11} = \begin{vmatrix} 9 & 4 \\ 2 & 8 \end{vmatrix} = 64$, $A_{12} = -\begin{vmatrix} 2 & 4 \\ 1 & 8 \end{vmatrix} = -12$, $A_{13} = \begin{vmatrix} 2 & 9 \\ 1 & 2 \end{vmatrix} = -5$

$A_{21} = -\begin{vmatrix} 4 & 2 \\ 2 & 8 \end{vmatrix} = -28$, $A_{22} = \begin{vmatrix} 8 & 2 \\ 1 & 8 \end{vmatrix} = 62$, $A_{23} = -\begin{vmatrix} 8 & 4 \\ 1 & 2 \end{vmatrix} = -12$

$A_{31} = \begin{vmatrix} 4 & 2 \\ 9 & 4 \end{vmatrix} = -2$, $A_{32} = -\begin{vmatrix} 8 & 2 \\ 2 & 4 \end{vmatrix} = -28$, $A_{33} = \begin{vmatrix} 8 & 4 \\ 2 & 9 \end{vmatrix} = 64$

(5) $\operatorname{adj} A = \begin{pmatrix} 64 & -12 & -5 \\ -28 & 62 & -12 \\ -2 & -28 & 64 \end{pmatrix}^T$

$$= \begin{pmatrix} 64 & -28 & -2 \\ -12 & 62 & -28 \\ -5 & -12 & 64 \end{pmatrix} \text{ Ans.}$$

$$\therefore A^{-1} = \frac{1}{|A|} \operatorname{adj} A = \frac{1}{454}\begin{bmatrix} 64 & -28 & -2 \\ -12 & 62 & -28 \\ -5 & -12 & 64 \end{bmatrix} \text{ Ans.}$$

HW Ex If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, দেখুওৱা যে $A^2 - 5A + 7I = 0$. A^{-1} উলিওৱা

~~Soln~~

Ex $A = \begin{pmatrix} 2 & -3 \\ 3 & 4 \end{pmatrix}$ যে $x^2 - 6x + 17 = 0$ সমীকৰণটো সিদ্ধ কৰে (দেখুওৱা)। A^{-1} ৰ মান নিৰ্ণয় কৰা।

Soln. $A^2 = AA = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 4-9 & -6-12 \\ 6+12 & -9+16 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -18 \\ 18 & 7 \end{bmatrix}$$

$$-6A = (-6)\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix} = \begin{pmatrix} -12 & 18 \\ -18 & -24 \end{pmatrix}$$

$$17I = 17\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 17 & 0 \\ 0 & 17 \end{bmatrix}$$

$$A^2 - 6A + 17I = \begin{bmatrix} -5 & -18 \\ 18 & 7 \end{bmatrix} + \begin{bmatrix} -12 & 18 \\ -18 & -24 \end{bmatrix} + \begin{bmatrix} 17 & 0 \\ 0 & 17 \end{bmatrix}$$

$$= \begin{bmatrix} -5-12+17 & -18+18+0 \\ 18-18+0 & 7-24+17 \end{bmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= 0$$

এতিয়া, $A^2 - 6A + 17I = 0$

$\Rightarrow A^2 - 6A = -17I$

$\Rightarrow A - 6I = -17A^{-1}$

$\Rightarrow -\frac{1}{17}\{A - 6I\} = A^{-1}$

$\Rightarrow -\frac{1}{17}\left\{\begin{pmatrix} 2 & -3 \\ 3 & 4 \end{pmatrix} - 6\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right\} = A^{-1}$

$\Rightarrow -\frac{1}{17}\begin{bmatrix} 2-6 & -3-0 \\ 3-0 & 4-6 \end{bmatrix} = A^{-1}$

$\Rightarrow -\frac{1}{17}\begin{pmatrix} -4 & -3 \\ 3 & -2 \end{pmatrix} = A^{-1}$

$\Rightarrow \begin{pmatrix} \frac{4}{17} & \frac{3}{17} \\ -\frac{3}{17} & \frac{2}{17} \end{pmatrix}$ Ans.

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5. ধৰা হ'ল $A = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$, $A^2 + aA + bI = 0$ হ'লে a, b আৰু A^{-1} উলিওৱা।

Sol^n $A^2 = AA = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 11 & 8 \\ 4 & 3 \end{pmatrix}$

এতিয়া, $A^2 + aA + bI = 0$

$$\begin{pmatrix} 11 & 8 \\ 4 & 3 \end{pmatrix} + a\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} + b\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 11+3a+b & 8+2a \\ 4+a & 3+a+b \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\Rightarrow 11+3a+b=0,\ 8+2a=0,\ 4+a=0\ \&\ 3+a+b=0$

$\Rightarrow a=-4$ and $b=1$

$\therefore A^2-4A+I=0$

$\Rightarrow 4A-A^2=I$

$\Rightarrow A(4I-A)=I$

$\Rightarrow A^{-1}=4I-A$

$$=\begin{pmatrix}4&0\\0&4\end{pmatrix}-\begin{pmatrix}3&2\\1&1\end{pmatrix}$$

$$=\begin{pmatrix}1&-2\\-1&3\end{pmatrix}$$ Ans.

Ex $A=\begin{pmatrix}1&2&2\\2&1&2\\2&2&1\end{pmatrix}$ মৌলকওৱাই $A^2-4A-5I_3=0$ সমীকৰণটো সিদ্ধ কৰিলে A^{-1} উলিওৱা।

Soln. $A^2=\begin{pmatrix}1&2&2\\2&1&2\\2&2&1\end{pmatrix}\begin{pmatrix}1&2&2\\2&1&2\\2&2&1\end{pmatrix}=\begin{pmatrix}9&8&8\\8&9&8\\8&8&9\end{pmatrix}$

$4A=\begin{pmatrix}4&8&8\\8&4&8\\8&8&4\end{pmatrix}$ $\quad 5I=\begin{pmatrix}5&0&0\\0&5&0\\0&0&5\end{pmatrix}$

$A^2-4A-5I_3=0$

$\Rightarrow A^2-4A=5I$

$\Rightarrow A-4I=5A^{-1}$ (উভয় পক্ষক A^{-1} ৰে পূৰণ কৰি)

$\Rightarrow A^{-1}=\frac{1}{5}\{A-4I\}$

$$=\frac{1}{5}\left\{\begin{pmatrix}1&2&2\\2&1&2\\2&2&1\end{pmatrix}-\begin{pmatrix}4&0&0\\0&4&0\\0&0&4\end{pmatrix}\right\}$$

$$=\frac{1}{5}\begin{bmatrix}-3&2&2\\2&-3&2\\2&2&-3\end{bmatrix}$$

$$=\begin{bmatrix}-3/5&2/5&2/5\\2/5&-3/5&2/5\\2/5&2/5&-3/5\end{bmatrix}$$ Ans